GABARITO - $2^a$ PROVA

FÍSICA 3 - Escola Politécnica - 2000

FGE 2295 - 16/5/2000

# Questão 1

a) $\vec{E} = \frac{Q}{4\pi\kappa\varepsilon_0 r^2}\,\hat{r}$ (Gauss)

$-V = -\int_a^b \vec{E}\cdot d\vec{s} = -\int_a^b \frac{Q}{4\pi\kappa\varepsilon_0 r^2}\,dr$

$\Rightarrow V = \frac{-Q}{4\pi\kappa\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)$

$Q = 4\pi\varepsilon_0\kappa V\left(\frac{1}{a} - \frac{1}{b}\right)^{-1} = 4\pi\kappa\varepsilon_0 V\left(\frac{ab}{b-a}\right)$

b) $\vec{E} = \frac{Q}{4\pi\kappa\varepsilon_0 r^2}\,\hat{r}\,;\quad \vec{D} = \kappa\varepsilon_0\vec{E} = \frac{Q}{4\pi r^2}\,\hat{r}$

$\vec{D} = \varepsilon_0\vec{E} + \vec{P} \quad \Rightarrow \quad \vec{P} = \frac{Q}{4\pi r^2}\left(1 - \frac{1}{\kappa}\right)\hat{r} = \frac{Q}{4\pi r^2}\left(\frac{\kappa - 1}{\kappa}\right)\hat{r}$

c) $C = \frac{Q}{V} = 4\pi\kappa E\varepsilon_0\left(\frac{ab}{b-a}\right)$

$U = \frac{1}{2}CV^2 = 2\pi\kappa\varepsilon_0 V^2\left(\frac{ab}{b-a}\right)$

# Questão 2

a) A distância r do eixo: área = $2\pi r\,h$

$\vec{J} = \frac{I_0}{2\pi r\,h}\,\hat{r}$

b) $\vec{J} = \sigma\vec{E} \quad \Rightarrow \quad \vec{E} = \frac{I_0}{2\pi\sigma h r}\,\hat{r}$

$V_{ab} = -\int_a^b \vec{E}\cdot d\vec{s} = -\int_a^b \frac{I_0}{2\pi\sigma h}\frac{dr}{r} = \frac{I_0}{2\pi\sigma h}\,\ell n\left(\frac{a}{b}\right) < 0$

$V_{ba} = -V_{ab} = \frac{I_0}{2\pi\sigma h}\,\ell n\left(\frac{b}{a}\right)$

c) $V = RI_0 \quad \Rightarrow \quad R = \frac{V}{I_0} = \frac{\ell n\left(\frac{b}{a}\right)}{2\pi\sigma h}$

# Questão 3

a) $\vec{F}_1 = I_0\,\vec{\ell}_1 \times \vec{B} = I_0 a\, B_0(\,\hat{e}_y) \times \hat{e}_z$

$= I_0 a\, B_0\, \hat{e}_x$

$\vec{F}_2 = I_0\,\vec{\ell}_2 \times \vec{B} = -I_0 B_0 (a \cos 60^o \hat{e}_y + a\text{sen}60^o \hat{e}_z) \times \hat{e}_z$

$= -I_0\, B_0\, \frac{a}{2}(\hat{e}_y \times \hat{e}_z) = -\,I_0\, \frac{a}{2}\, B_0\, \hat{e}_x$

$\vec{B}$ uniforme, espira fechada $\Rightarrow \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$

$\Rightarrow \quad \vec{F}_3 = -\vec{F}_1 - \vec{F}_2 = -I_0\, \frac{a}{2}\, B_0 \hat{e}_x$

b) $\vec{\mu} = I\, \vec{A} = +\, I_0 A\, \hat{e}_x$

$A = \frac{a \cdot \frac{\sqrt{3}}{2}\, a}{2} = \frac{\sqrt{3}}{4}\, a^2$

$\Rightarrow \quad \vec{\mu} = \frac{\sqrt{3}}{4}\, I_0\, a^2\, \hat{e}_x$

c) $\vec{\tau} = \vec{\mu} \times \vec{B} = \frac{\sqrt{3}}{4}\, I_0\, a^2\, B_0\, (\hat{e}_x) \times \hat{e}_z = -\frac{\sqrt{3}}{4}\, I_0\, a^2\, B_0\, \hat{e}_y$

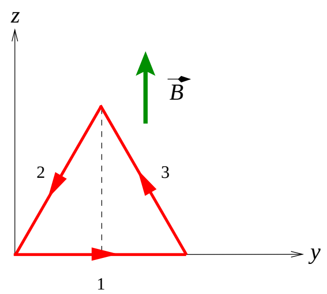


Figura 1:

## Questão 4

a) $\oint \vec{B} \cdot d\vec{s} = \mu_0 I$

$B \cdot 2\pi r = \mu_0 I \Rightarrow \vec{B} = \frac{\mu_0 I}{2\pi r}\,\hat{\theta}$

b) $d\vec{B} \;=\; \frac{\mu_0 I}{4\pi}\,\frac{d\vec{s}\times\hat{r}}{r^2}$

$d\vec{s}\times\hat{r} \;=\; \hat{e}_z \cdot ds$

$d\vec{B} \;=\; \frac{\mu_0 I}{4\pi}\,\frac{ds}{r^2}\,\hat{e}_z$

$\vec{B} \;=\; \frac{\mu_0 I}{4\pi}\,\frac{2\pi R}{R^2}\,\hat{e}_z \;=\; \frac{\mu_0 I}{2R}\,\hat{e}_z$

c) Princípio de Superposição: cálculo separado de semi-circunferência e segmentos retos.

$\vec{B}_2 \;=\; \vec{B}_4 \;=\; 0 \quad (d\vec{s}\,//\,\hat{r})$

$\vec{B}_1 \;=\; -\,\vec{B}_5 \quad \text{(Simetria)}$

$\vec{B} = \vec{B}_3 = \frac{\mu_0 I}{4\pi}\cdot\frac{\pi R}{R^2}\,\hat{e}_z \quad \Rightarrow \quad \vec{B} = \frac{\mu_0 I}{4R}\,\hat{e}_z$

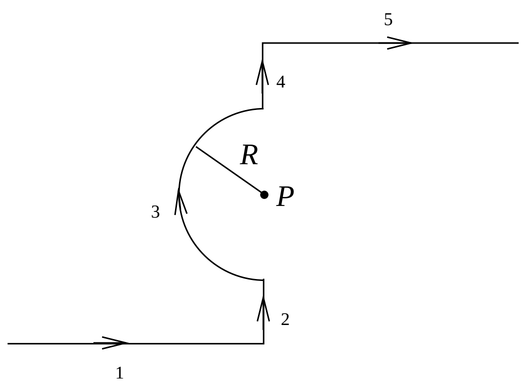


Figura 2: